

Heavy Quarkonium Melting in Large N Thermal QCD

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ABSTRACT: Large N QCD is mostly governed by planar diagrams and should show linear confinement when these diagrams are suitably summed. The linear confinement of quarks in a class of these theories using gravity duals that capture the logarithmic runnings of the coupling constants in the IR and strongly coupled asymptotic conformal behavior in the UV was studied in our previous work. We also extended the theories to high temperatures and argued the possibilities of meltings and suppressions of heavy quarkonium states. In this paper we give a formal proof of melting using very generic choices of UV completions, and point out some subtleties associated with meltings in generic large N theories. Our proof requires only the existence of well defined UV behaviors that are devoid of Landau poles and UV divergences of the Wilson loops, allowing degrees of freedom to increase monotonously with energy scale. We determine the melting temperatures of heavy quarkonium states, which could suggest the presence of deconfinement phase transitions in these theories.

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1. Introduction

Large N QCD in $3 + 1$ dimension is a relatively simpler theory than its finite N counterpart, owing in part to the $1/N$ suppressions of the non-planar diagrams, but its solution is still a challenge. For example its not clear how to add up all the planar diagrams and argue for linear confinement of fundamental quarks. Although in terms of mesonic and glue-ball degrees of freedom the theory may look *free*, this is an oversimplification. Owing to the existence of a Hagedorn temperature and, at a more fundamental level, quark degrees of freedom, the system is still complex with no simple way of computing, for example, the current-current correlator or the master field [1].

In principle, gauge-gravity duality provides a way to compute - or at least allow some analytic control on - some of these quantities. However the problem is that to restrict everything to the *supergravity* level, where we can have more analytic control, the gauge theory should be at strong 'tHooft coupling. This cannot happen for large N QCD that is asymptotically free (although there could be a full string theory dual description). If one relaxes that condition and looks for strongly coupled asymptotic conformal behavior then one can construct a supergravity dual for a class of these theories.

In a previous paper [3], which was a continuation of earlier work [2], a new supergravity background that captures the logarithmic runnings of the coupling constants of a particular class of large N QCD in the far IR and the strongly coupled asymptotic conformal behavior in the far UV was constructed. In the intermediate energy scales, our dual gravitational background captures the interpolating behavior of the beta function.

In [3] it was argued that such a geometry would consist of three regions termed region 1, 2 and 3 that would capture the IR, the intermediate scale, and the UV respectively. The supergravity solution was constructed using fluxes sourced by N number of D3 branes, M number of D5 branes and anti-branes respectively while taking the back reaction of N_f number of seven branes and $N_f - 3$ number of anti seven branes. The metric in all three regions can be written in the following form [3]

$$ds^2 = \frac{1}{\sqrt{h}} \left[-g_1 dt^2 + dx^2 + dy^2 + dz^2 \right] + \sqrt{h} \left[g_2^{-1} g_{rr} dr^2 + g_{mn} dx^m dx^n \right] \quad (1.1)$$

with g_i being the black-hole factors and h being the warp factor depending on all the internal coordinates $(r, \theta_i, \phi_i, \psi)$. To zeroth order in $g_s N_f$ and $g_s M$ we have our usual relations:

$$\begin{aligned} h^{[0]} &= \frac{L^4}{r^4}, & g^{[0]} &= 1 - \frac{r_h^4}{r^4}, & g_{rr}^{[0]} &= 1, & g_{mn}^{[0]} dx^m dx^n &= ds_{T^{11}}^2 \\ g_1^{[0]} = g_2^{[0]} &\equiv g = 1 - \frac{r_h^4}{r^4}, & L^4 &\equiv g_s N \alpha'^2 \end{aligned} \quad (1.2)$$

where T^{11} is the base of a six dimensional conifold and it has the topology of $S^3 \times S^2$, where S^3 and S^2 are three-sphere and two-sphere. Here g_s is the string coupling, L is the AdS throat radius and r_h is the black hole horizon. The above metric is that of a ten dimensional geometry with coordinates (t, x, y, z, r) describing a five dimensional non compact space while internal coordinates $(\theta_1, \theta_2, \phi_1, \phi_2, \psi)$ describe a five dimensional compact space.

At higher orders in $g_s N_f, g_s M$, the warp factor, the black hole factors¹ and the internal metric get modified because of the back-reactions from the seven-branes, three form fluxes and the localized sources we embed. We can write this as:

$$h = h^{[0]} + h^{[1]}, \quad g_{rr} = g_{rr}^{[0]} + g_{rr}^{[1]}, \quad g_{mn} = g_{mn}^{[0]} + g_{mn}^{[1]}, \quad g_i = g_i^{[0]} + g_i^{[1]} \quad (1.3)$$

where the superscripts denote the order of $g_s N_f$ and $g_s M$.

In regions 1 and 2, the warp factor h has logarithmic dependence while in region 3, it can be expanded exclusively as a power series in $1/r \equiv u$. In terms u coordinate, we have the following Taylor series expansion for the warp factor h :

$$\frac{1}{\sqrt{h}} = \mathcal{A}_n u^{n-2} \quad (1.4)$$

where \mathcal{A}_n are Taylor coefficients and in general can be functions of internal coordinates. Region 3 was taken to be large enough so that the Nambu-Goto string would

¹This also means that the black hole horizon is no longer the surface $r = r_h$ and the equation for the surface may be more involved.

lie completely inside it. The computation of the potential at zero temperature revealed that at short distances the potential should be dominated by the Coulomb term²[3]:

$$V_{Q\bar{Q}} = -\frac{0.236}{d} + \mathcal{O}(d) \quad (1.5)$$

where the numerical value and the sign of the first term are fixed naturally by the dual background. It is interesting to note that this value is of the same order of magnitude as the one derived from QCD lattice simulations [5]. For large distances one expects the potential to be dominated by the linear term [3]:

$$V_{Q\bar{Q}} = \left(\frac{\mathcal{A}_n x_{\max}^n}{\pi x_{\max}^2} \right) d \quad (1.6)$$

where x_{\max} is the maximum value for the depth of the U shaped string denoted by u_{\max} and it is a solution of the constraint equation (see section 2 and also [3] for details):

$$\frac{1}{2}(m+1)\mathcal{A}_{m+3}x_{\max}^{m+3} = 1 \quad (1.7)$$

The constraint equation is obtained by demanding that the distance between the quarks and the potential energy of the quark pair is real. For AdS space, interquark distance d and potential $V_{Q\bar{Q}}$ is *always* real for any value of u_{\max} , and hence x_{\max} do not exist, as there is *no* constraining equation.

At high temperatures and density, medium effects should *screen* the interaction between the heavy quark the anti-quark. The resulting effective potential between the quark anti-quark pairs separated by a distance d at temperature T can then be expressed succinctly in terms of the free energy $F(d, T)$, which generically takes the following form:

$$F(d, T) = \sigma d f_s(d, T) - \frac{\alpha}{d} f_c(d, T) \quad (1.8)$$

where σ is the string tension, α is the gauge coupling and f_c and f_s are the screening functions³ (see for example [6] and references therein). For the quark and the anti-quark pair kept at respective positions of $+\frac{d}{2}$ and $-\frac{d}{2}$, we expect the Wilson line $W(\pm\frac{d}{2})$ to be related to the free energy through

$$\exp \left[-\frac{F(d, T)}{T} \right] = \frac{\langle W^\dagger(+\frac{d}{2}) W(-\frac{d}{2}) \rangle}{\langle W^\dagger(+\frac{d}{2}) \rangle \langle W(-\frac{d}{2}) \rangle} \quad (1.9)$$

²In deriving the Coulomb potential, we have set AdS throat radius $L \equiv 1$ and string tension $T_0 \equiv 1$ for convenience, and this is only a redefinition of units. By restoring units, one obtains $V_{Q\bar{Q}} = -\frac{0.236}{d} \frac{\sqrt{g_s N}}{d} + \mathcal{O}(d)$, as expected [4].

³We expect the screening functions f_s, f_c to equal identity when the temperature goes to zero, yielding the zero temperature Cornell potential.

In terms of Wilson loop, the free energy Eq.(1.8) is now related to the renormalised Nambu-Goto action for the string on a background with a black-hole⁴. One may also note that the final theory is not three-dimensional, but four-dimensional and *compactified* on a circle in Euclideanised version.

Using Eq.(1.9) and the identification of the Wilson loop to the Nambu-Goto action, the free energy (or equivalently the potential) between the heavy quark and the anti-quark at non-zero temperatures can be deduced. For large distances the potential is [3]

$$V_{Q\bar{Q}} = \sqrt{1 - \frac{y_{\max}^4}{u_h^4}} \left(\frac{\mathcal{A}_n y_{\max}^n}{\pi y_{\max}^2} \right) d \quad (1.10)$$

where y_{\max} is the maximum value for the depth u_{\max} of the U shaped string in the presence of a black hole. It is given by solving the constraint equation:

$$\frac{1}{2}(m+1)\mathcal{A}_{m+3}y_{\max}^{m+3} + \frac{1}{j!} \prod_{k=0}^{j-1} \left(k - \frac{1}{2} \right) \left(\frac{y_{\max}^4}{u_h^4} \right)^j \left[\mathcal{A}_l y_{\max}^l \left(\frac{l}{2} + 2j - 1 \right) \right] = 1 \quad (1.11)$$

which arises by demanding that inter quark separation and free energy is always real [3]. Note that $u_h = 1/r_h$ and for zero temperature, $u_h = \infty$. Thus at zero temperature, $u_h \rightarrow \infty$ and (1.11) reduces to (1.7).

At this point, it is important to define in a more precise manner what exactly is meant by melting. In this work, “melting” is meant to quantify the disappearance of the linear portion of the quark-antiquark potential, at a given length scale. There is therefore a qualitative difference between this behavior and the actual dissolution of quarkonium bound states: The robustness of the $Q\bar{Q}$ pairs will actually depend on *where* their energy sits in the potential profile. Later studies could also involve a more precise characterization through a study of the temperature dependence of quarkonium spectral densities, for example.

Going back to Eq.(1.10), one might think that the melting temperature u_c^{-1} is given by the condition that $y_{\max} = u_h$ is a solution of Eq.(1.11) so that the coefficient of d in Eq.(1.10) vanishes. Actually, this turned out *not* to be the case. As shown below, Eq.(1.11) can never have a solution at $y_{\max} = u_h$. Hence, the deconfinement in our case does not mean a complete absence of the linear potential. Rather, it means that the linear potential ceases to have an infinite range and as temperature increases, the range of the linear potential quickly becomes short provided that Eq.(1.11) allows

⁴There is a large body of literature on the subject where quark anti-quark potential has been computed using various different approaches like [7]. Although the potential Eq.(1.10) that we get matches well with other results, we have an additional constraint: Eq.(1.11), from the background RG flow. This additional constraint, which cannot be seen from an AdS or an AdS with an IR cut-off dual, will have non-trivial consequences for melting that we will discuss in section 2.

a real positive solution lying in region 3. It is not clear this would always be the case, so Eq.(1.11) requires a more detailed investigation. In the following section i.e sec. 2, we will carefully analyse Eq.(1.11) for generic choices of warp factors or more appropriately, generic UV completions that have no Landau poles or UV divergences of the Wilson loops, and give a proof of quarkonium melting for this class of theories. Section 3 contains a detailed numerical analysis that will allow us to find the melting points, and could also suggest the presence of a deconfinement phase transition in these theories. We conclude with a short discussion.

2. Proof of the existence of a melting point

Our cascading picture of renormalization group flow demands that in the region 3, the effective number of colors grows as r grows. The number of colors at any scale $u = 1/r$ in the region 3 is given by $N_{\text{eff}}(u) = N(1 + a_l u^l)$. For the analysis given here, it is simpler to define and use

$$\mathcal{F}(u) \equiv \frac{u^2}{\sqrt{h}} = \frac{\sqrt{N}}{L^2 \sqrt{N_{\text{eff}}}} = \frac{1}{L^2 \sqrt{1 + a_l u^l}} \quad (2.1)$$

instead of $N_{\text{eff}}(u)$. The coefficients \mathcal{A}_n appearing in Eqs. (1.4), (1.7) and (1.11) are related to $\mathcal{F}(u)$ by $\mathcal{F}(u) = \mathcal{A}_n u^n$; and h is the warp factor. In terms of $\mathcal{F}(u)$, the condition that $N_{\text{eff}}(u)$ is a decreasing function of $u = 1/r$ becomes

$$\mathcal{F}'(u) > 0 \quad (2.2)$$

Combining Eqs.(2.1) and (2.2) yields the following condition

$$\mathcal{F}(u) > \frac{1}{L^2} \quad (2.3)$$

From now on, and as mentioned earlier, the value of L is set to 1 so that $\mathcal{F}(u) > 1$.

2.1 Zero temperature

Let u_{max} be the maximum value of u for the string between the quark and the anti-quark. Then the relationship between u_{max} and the distance between the quark and the anti-quark is given by [3]

$$d(u_{\text{max}}) = 2u_{\text{max}}\mathcal{F}(u_{\text{max}}) \int_{\epsilon_0}^1 dv \frac{v^2 \sqrt{\mathcal{G}_m u_{\text{max}}^m v^m}}{(\mathcal{F}(u_{\text{max}}v))^2} \left[1 - v^4 \left(\frac{\mathcal{F}(u_{\text{max}})}{\mathcal{F}(u_{\text{max}}v)} \right)^2 \right]^{-1/2} \quad (2.4)$$

Two conditions must be met before asserting that this expression represents the physical distance between a quark and an anti-quark in vacuum. One obvious condition is that the integral is real. This is guaranteed if for all $0 \leq v \leq 1$:

$$W(v|u_{\text{max}}) \equiv v^2 \left(\frac{\mathcal{F}(u_{\text{max}})}{\mathcal{F}(u_{\text{max}}v)} \right) \leq 1 \quad (2.5)$$

Another condition is that the potential between the quark and the anti-quark must be long ranged. That is, $d(u_{\max})$ must range from 0 to ∞ as u_{\max} varies from 0 to some finite value, say $u_{\max} = x_{\max}$. Since $\mathcal{F}(u) > 1$, the only way to satisfy these conditions is via the (sufficiently fast) vanishing of the square-root in Eq.(2.4) as $v \rightarrow 1$ at $u_{\max} = x_{\max}$.

For most u_{\max} , $1 - W(v|u_{\max})^2$ vanishes only linearly as v approaches 1. In this case, $d(u_{\max})$ is finite since the singularity in the integrand behaves like $1/\sqrt{1-v}$ and hence integrable. To make $d(u_{\max})$ diverge at $u_{\max} = x_{\max}$, $1 - W(v|x_{\max})^2$ must vanish quadratically as v approaches 1 so that the integrand is sufficiently singular, $1/\sqrt{1 - W(v|x_{\max})^2} \sim 1/|1 - v|$. Therefore, the function $W(v|x_{\max})$ must have a maximum at $v = 1$.

To determine the value of x_{\max} , consider

$$W'(v|x_{\max}) = 2v \left(\frac{\mathcal{F}(x_{\max})}{\mathcal{F}(x_{\max}v)} \right) \left(1 - (x_{\max}v) \frac{\mathcal{F}'(x_{\max}v)}{2\mathcal{F}(x_{\max}v)} \right) \quad (2.6)$$

For this to vanish at $v = 1$, x_{\max} must be the smallest positive solution of

$$x\mathcal{F}'(x) - 2\mathcal{F}(x) = 0 \quad (2.7)$$

With the definition $\mathcal{F}(u) = \mathcal{A}_n u^n$, one can easily show that this is equivalent to the condition (1.7) which was originally derived in [3]. The allowed range of u_{\max} is then

$$0 \leq u_{\max} \leq x_{\max} \quad (2.8)$$

and within this range, $d(u_{\max})$ varies from 0 to ∞ . How it varies will depend on the values of \mathcal{G}_m as well as $\mathcal{F}(u)$.

2.2 Finite temperature

At finite temperature, the relation between u_{\max} and the distance between the quark and the anti-quark is obtained by replacing $\mathcal{F}(u)$ with $\sqrt{1 - u^4/u_h^4} \mathcal{F}(u)$ in Eq.(2.4):

$$\begin{aligned} d_T(u_{\max}) &= 2u_{\max} \sqrt{1 - u_{\max}^4/u_h^4} \mathcal{F}(u_{\max}) \int_{\epsilon_0}^1 dv \frac{v^2 \sqrt{\mathcal{D}_m u_{\max}^m v^m}}{(1 - v^4 u_{\max}^4/u_h^4)(\mathcal{F}(u_{\max}v))^2} \\ &\times \left[1 - v^4 \frac{(1 - u_{\max}^4/u_h^4)}{(1 - v^4 u_{\max}^4/u_h^4)} \left(\frac{\mathcal{F}(u_{\max})}{\mathcal{F}(u_{\max}v)} \right)^2 \right]^{-1/2} \end{aligned} \quad (2.9)$$

The explicit factor of u_{\max} makes $d_T(u_{\max})$ vanish at $u_{\max} = 0$ as in the $T = 0$ case. As u_{\max} approaches u_h , the integral near $v = 1$ behaves like

$$d_T(u_{\max}) \sim \int_0^1 dv \frac{\sqrt{1 - u_{\max}^4/u_h^4}}{\sqrt{(1-v)(1 - v u_{\max}/u_h)}} \quad (2.10)$$

which indicates that $d_T(u_{\max})$ goes to 0 as u_{\max} approaches u_h . Hence, at both $u_{\max} = 0$ and $u_{\max} = u_h$, $d_T(u_{\max})$ vanishes. Since $d_T(u_{\max})$ is positive in general, there has

to be a maximum between $u_{\max} = 0$ and $u_{\max} = u_h$. Whether the maximum value of $d_T(u_{\max})$ is infinite as in the $T = 0$ case depends on the temperature (equivalently, u_h^{-1}) as we now show.

The fact that the physical distance needs to be real yields the following condition. For all $0 \leq v \leq 1$,

$$W_T(v|u_{\max}) \equiv v^2 \left(\frac{\mathcal{F}(u_{\max})}{\mathcal{F}(u_{\max}v)} \right) \sqrt{\frac{1 - u_{\max}^4/u_h^4}{1 - u_{\max}^4 v^4/u_h^4}} \leq 1 \quad (2.11)$$

Taking the derivative gives

$$\begin{aligned} W'_T(v|u_{\max}) &= \frac{(1 - u_{\max}/u_h^4)^{1/2}}{(1 - u_{\max}^4 v^4/u_h^4)^{3/2}} \frac{v\mathcal{F}(u_{\max})}{\mathcal{F}(u_{\max}v)} \\ &\quad \times [-(u_{\max}v)(1 - (u_{\max}v/u_h)^4)\mathcal{F}'(u_{\max}v) + 2\mathcal{F}(u_{\max}v)] \end{aligned} \quad (2.12)$$

Similarly to the $T = 0$ case, $d_T(u_{\max})$ can have an infinite range if the derivative vanishes at $v = 1$ for a certain value of u_{\max} , say $u_{\max} = y_{\max}$. This value of y_{\max} is determined by the smallest positive solution of the following equation

$$y\mathcal{F}'(y) - 2\mathcal{F}(y) = (y/u_h)^4 y\mathcal{F}'(y) \quad (2.13)$$

which then forces $W'_T(1|y_{\max})$ to vanish. Note that the left hand side is the same as the zero temperature condition, Eq.(2.7). The right hand side is the temperature (u_h) dependent part. Using the facts that:

$$\prod_{k=0}^{j-1} (k - 1/2) = -(2j - 3)!!/2^j, \quad \sum_{j=1}^{\infty} x^j (2j - 3)!!/2^j j! = 1 - \sqrt{1 - x}, \quad (2.14)$$

it can be readily shown that Eq.(2.13) is equivalent to Eq.(1.11) as long as $u_{\max} < u_h$. It is also clear that $y = u_h$ cannot be a solution of Eq.(2.13) because at $y = u_h$, the equation reduces to $\mathcal{F}(u_h) = 0$ which is inconsistent with the fact that $\mathcal{F}(y) \geq 1$.

Recall that $\mathcal{F}(y) \geq 1$ and $\mathcal{F}'(y) \geq 0$ in the region 3 and we assume that the equation $y\mathcal{F}'(y) - 2\mathcal{F}(y) = 0$ has a real positive solution x_{\max} . Hence as y increases from 0 towards x_{\max} , the left hand side of Eq.(2.13) increases from -2 while the right hand side increases from 0. The left hand side reaches 0 when $y = x_{\max}$ which is the point where the distance $d(u_{\max})$ at $T = 0$ becomes infinite. At this point the right hand side of Eq.(2.13) is positive and has the value $(x_{\max}/u_h)^4 x_{\max} \mathcal{F}'(x_{\max})$. Hence the solution of Eq.(2.13), if it exists, must be larger than x_{\max} .

Consider first low enough temperatures so that $u_h \gg x_{\max}$. For these low temperatures, Eq.(2.13) will have a solution, as the right hand side will be still small around $y = x_{\max}$. This then implies that the linear potential at low temperature will have an infinite range if the zero temperature potential has an infinite range.

To show that the infinite range potential cannot be maintained at all temperatures, let $u_h = x_{\max}$. When the left hand side vanishes at $y = x_{\max}$, the right hand side is $x_{\max} \mathcal{F}'(x_{\max}) = 2\mathcal{F}(x_{\max})$ which is positive and finite. For $y > x_{\max}$, the left hand side $(y\mathcal{F}'(y) - 2\mathcal{F}(y))$ may become positive, but it is always smaller than $y\mathcal{F}'(y)$ since $\mathcal{F}(y)$ is always positive. But for the same y , the right hand side $((y/u_h)^4 y\mathcal{F}'(y))$ is always positive and necessarily larger than $y\mathcal{F}'(y)$ since $(y/u_h) > 1$. Hence, Eq.(2.13) cannot have a real and positive solution when $u_h = x_{\max}$. Therefore between $u_h = \infty$ and $u_h = x_{\max}$, there must be a point when Eq.(2.13) cease to have a positive solution.

When Eq.(2.13) has no solution, then the expression for $d_T(u_{\max})$, (2.9) will not diverge for any u_{\max} within $(0, u_h)$. Furthermore, since the expression vanishes at both ends, there must be a maximum $d_T(u_{\max})$ at a non-zero u_{\max} . When the distance between the quark and the anti-quark is greater than this maximum distance, there can no longer be a string connecting the quark and the anti-quark.

3. Numerical analyses of melting temperatures

After discussing the most general choice for warp factors that give rise to y_{\max} and consequently linear potential, specific examples of geometries that may arise as solutions to Einstein's equation will be considered, starting with the following ansatz for the metric:

$$\begin{aligned} ds^2 &= -\frac{g}{\sqrt{h}} dt^2 + \frac{1}{\sqrt{h}} (dx^2 + dy^2 + dz^2) + \frac{\sqrt{h}}{u^2} \left(\frac{H}{gu^2} du^2 + ds_{\mathcal{M}_5}^2 \right) \\ &\equiv -\frac{g}{\sqrt{h}} dt^2 + \frac{1}{\sqrt{h}} (dx^2 + dy^2 + dz^2) + \frac{\sqrt{h}}{u^2} \tilde{g}_{mn} dx^m dx^n \end{aligned} \quad (3.1)$$

where $h \equiv h(u, \theta_i, \phi_i, \psi)$, $H \equiv H(u, \theta_i, \phi_i, \psi)$, and $g \equiv 1 - u^4/u_h^4$; \mathcal{M}_5 is the compact five dimensional manifold parametrised by coordinates (θ_i, ϕ_i, ψ) and can be thought of as a perturbation over $T^{1,1}$. Here $u = 0$ is the boundary and $u = u_h$ is the horizon. As discussed in earlier work [3], the above metric arises in region 3 of [3] when one considers the running of axio-dilaton τ , $D7$ brane local action and fluxes due to anti five-branes on a geometry that deviates from the IR OKS-BH geometry from the backreactions of the above sources. The three-form fluxes sourced by (p, q) anti-branes are proportional to $r^{-i} f(r)$ for some positive i (see [3] for details about $f(r)$), where the function $f(r) \rightarrow 1$ as $r \rightarrow \infty$ and $f(r) \rightarrow 0$ as $r \rightarrow 0$. With the coordinate $u = 1/r$, there is another function: $k(u) \equiv \exp(-u^A)$, $A > 0$, that also has somewhat similar behaviour as $f(u)$ and may allow us to have a better analytic control on the background. With such a choice of $k(u)$, the total three from flux is proportional to $u^A M(u)$ with

$$M(u) \equiv M[1 - k(u)] = M[1 - \exp(-u^A)] \quad (3.2)$$

where M is the number of bi-fundamental flavors. Thus three-form fluxes are decaying fast as $Mu^A [1 - \exp(-u^A)]$ and, as shown in [3], the seven-branes could be arranged such that the axio-dilaton τ behaves typically as $\tau \sim u^B$. This means that from the behavior of the internal Riemann tensor one may conclude that the internal metric \tilde{g}_{mn} behaves as $\tilde{g}_{mn} \sim u^C \exp(c_o u^C)$ where A, C, \mathcal{A} and \mathcal{C} are all positive and c_o could be positive or negative depending on the precise background informations.

From the above discussions it should be clear that taking the three-forms and world-volume gauge fluxes to be exponentially decaying in the IR (but axio-dilaton to be suppressed only as u^B) should solve all the equations of motion, giving the following behavior for the warp factor h and the internal metric H in (3.1)⁵:

$$h = L^4 u^4 \exp(-\alpha u^{\tilde{\alpha}}), \quad H = \exp(\beta u^{\tilde{\beta}}) \quad (3.3)$$

where we are taking $\alpha, \tilde{\alpha}, \beta, \tilde{\beta}$ to be all positives with α, β to be functions of internal coordinates (θ_i, ϕ_i, ψ) and $L^4 = g_s N \alpha'^2$ to be the asymptotic AdS throat radius⁶.

Motivated by the above arguments, we will consider Nambu-Goto action of the string in the geometry with $(\tilde{\alpha}, \tilde{\beta}) = (3, 3)$ and $(\tilde{\alpha}, \tilde{\beta}) = (4, 4)$ at temperatures $T^{(1)}$ and $T^{(2)}$ respectively in Eq.(3.3). As in [3, 2] we consider mappings $X^\mu(\sigma, \tau)$, which are points in the internal space, to lie on the slice:

$$\theta_1 = \theta_2 = \pi, \quad \phi_i = 0, \quad \psi = 0 \quad (3.4)$$

so that on this slice α, β are fixed and we set it to $(\alpha, \beta) = (0.1, 0.05)$ for both choices $(\tilde{\alpha}, \tilde{\beta})$. (Such a choice of slice will also help us to ignore the three-form contributions to the Wilson loop.) With these fixed choices for the warp factors, we plot the interquark separation d as a function of u_{\max} in figures 1 and 2 for various values of $T \equiv 1/u_h$.

Note that for both choices of warp factors, for low enough temperatures, there exist $u_{\max} = y_{\max}$ where $d \rightarrow \infty$. As the temperature is increased, y_{\max} increases modestly. On the other hand from figure 3, one sees that when $T > T_c^{(1)} \sim 0.28$ there exists a d_{\max} which is finite. This means for interquark distance $d > d_{\max}$, there is *no* string configuration with boundary condition $x(0) = \pm d/2$ implying that the string attaching the quarks breaks and we have two *free* partons for $d > d_{\max}$. Thus we can interpret d_{\max} to be a “screening length”. From figure 2 we observe similar behaviour but now d_{\max} exists for $T > T_c^{(2)} \sim 0.399$.

In figure 3, d_{\max} as a function of T is plotted. We note that for a small change in the temperature near $T_c^{(1)}$ (or near $T_c^{(2)}$ equivalently) there is a sharp decrease in screening length d_{\max} , but for $T \gg T_c^{(i)}$, $i = 1, 2$, the screening length does not

⁵See also the interesting works of [8] where exponential warp factors have been chosen.

⁶Note that β in (3.3) could be considered negative so that H would be decaying to zero in the IR. However since region 3 doesn't extend to the IR we don't have to worry about the far IR behavior of Eq.(3.3).

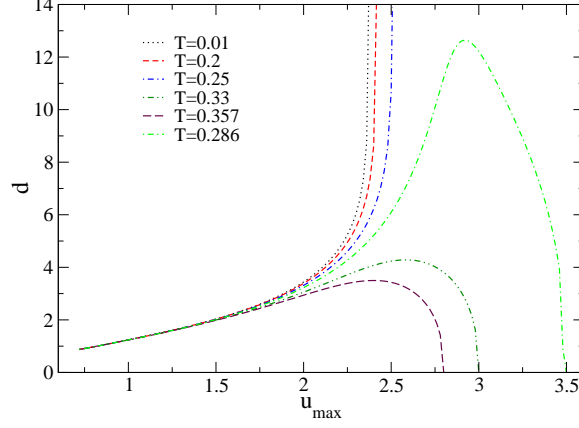


Figure 1: Interquark distance as a function of u_{\max} for various temperatures and warp factor with $(\alpha, \tilde{\alpha}, \beta, \tilde{\beta}) = (0.1, 3, 0.05, 3)$ in the warp factor equation.

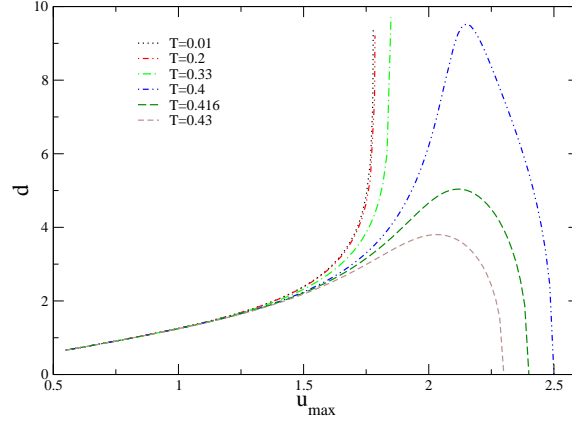


Figure 2: Quark-antiquark distance as a function of u_{\max} for various temperatures and warp factor with $(\alpha, \tilde{\alpha}, \beta, \tilde{\beta}) = (0.1, 4, 0.05, 4)$ in the warp factor equation.

change much. In fact d_{\max} behaves as $C + \exp(-\gamma T)$ (where C and γ are constants) which in turn could be an indicative of a phase transition near $T_c^{(i)}$ for $i = 1, 2$ i.e the two choices of warp factor.

Finally we plot the potential $V_{Q\bar{Q}}$ as a function of d in figures 4 and 5 for the two choices of warp factor. For $T < T_c^{(1)}$ in figure 4 and $T < T_c^{(2)}$ in figure 5, we have potential energies linearly increasing with an arbitrarily large increment of the interquark separations. Thus we have linear confinement of quarks for large distances and small enough temperatures. For $T > T_c^{(i)}$, $i = 1$ or 2 , there exists a d_{\max} and

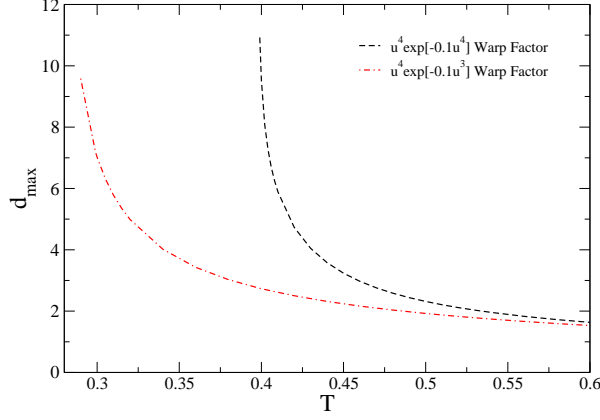


Figure 3: Maximum interquark separation d_{\max} as a function of $T = 1/u_h$ for both cubic and quartic warp factors.

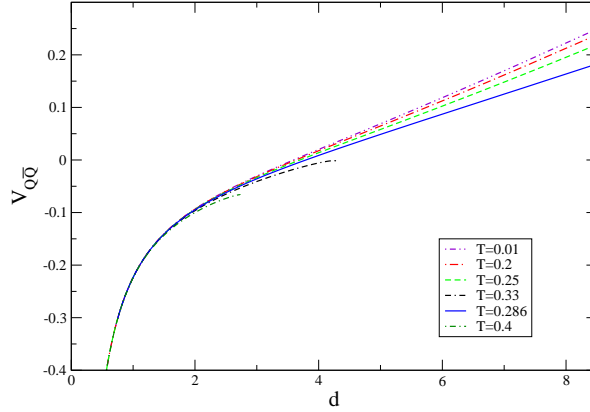


Figure 4: Heavy quark potential $V_{Q\bar{Q}}$ as a function of quark separation d with cubic warp factor, or equivalently, $(\alpha, \tilde{\alpha}, \beta, \tilde{\beta}) = (0.1, 3, 0.05, 3)$ in the warp factor equation for various temperatures. As mentioned in the text, one shouldn't consider the free energy (or equivalently the potential) to stop abruptly in the plot. After the string connecting the quarks breaks, the curves should be extrapolated by the contributions to the potential energy from the two free strings *and* their world-volume fluctuations for all $T > T_c^{(i)}$.

for all distances $d > d_{\max}$ there are no Nambu-Goto actions, S_{NG} , for the string attaching *both* the quarks. This means that we have free quarks and $V_{Q\bar{Q}}$ is constant for $d > d_{\max}$. Of course looking at figures 4 and 5 one shouldn't conclude that the free energy *stops* abruptly. What happens for those two cases is that the string joining

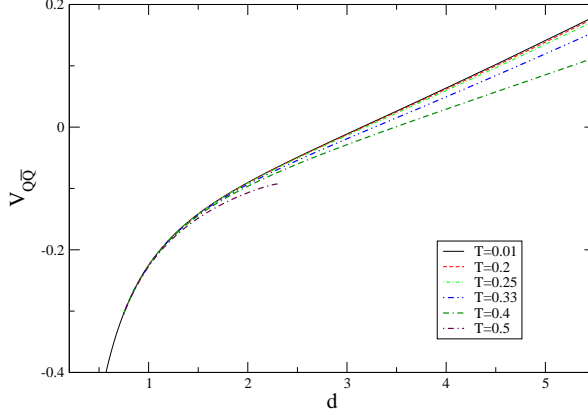


Figure 5: Heavy quark potential $V_{Q\bar{Q}}$ as a function of quark separation d with quartic warp factor, or equivalently, $(\alpha, \tilde{\alpha}, \beta, \tilde{\beta}) = (0.1, 4, 0.05, 4)$ in the warp factor equation for various temperatures. Here again, beyond $T > T_c$ the curves should be extrapolated by the contributions to the potential energy from the two free strings *and* their world-volume fluctuations.

the quarks breaks, and then the free energy is given by the sum of the energies of the two strings (from the tips of the seven-branes to the black-hole horizon) and the total energies of the small fluctuations on the world-volume of the strings. The latter contributions are non-trivial to compute and we will not address these in any more detail here, but energy conservations should tell us how to extrapolate the curves in figures 4 and 5, beyond the points where the string breaks, for all $T > T_c$. Of course after sufficiently long time the two strings would dissipate their energies associated with their world-volume fluctuations and settle down to their lowest energy states.

Observe that for a wide range of temperatures $0 < T < T_c^{(i)}$, the potential and thus the free energy hardly changes. But near a narrow range of temperatures $T_c^{(i)} - \epsilon < T < T_c^{(i)} + \epsilon$ (where $\epsilon \sim 0.05$), free energy changes significantly. For figure 5 the change is more abrupt than figure 4. This means as we go for bigger values of $\tilde{\alpha}$, the change in free energy is sharper.

In figure 6, we plot the slope of the linear potential as a function of T . Again for a wide range of temperatures, there are no significant changes in the slope but near $T_c^{(i)}$, the change is more dramatic: the slope decreases sharply, indicating again the possibility of a phase transition near $T_c^{(i)}$. As we noticed before, here too bigger exponent $\tilde{\alpha}$ gives a sharper decline in the slope hinting that when $\tilde{\alpha} \gg 1$, the transition would be more manifest.

To conclude, the above numerical analyses suggest the presence of a deconfinement transition, where for a narrow range of temperatures $0.28 \leq T_c \leq 0.39$ the free

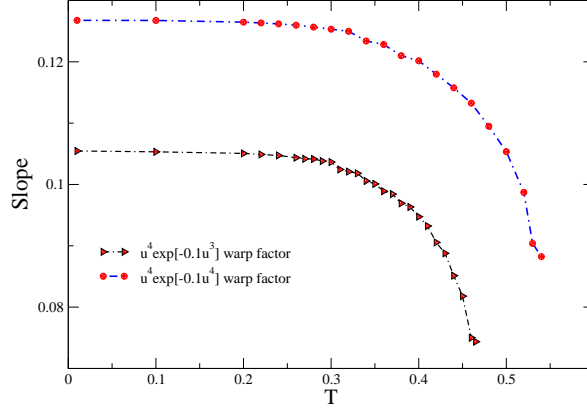


Figure 6: Slope of linear potential as a function of T for both cubic and quartic warp factors. Note that in the figure the slopes have been computed as $\frac{\Delta V}{\Delta d}$ with a range of d from 1.6 to 1.7 in appropriate units.

energy of $Q\bar{Q}$ pair shows a sharp decline. Interestingly, changing the powers of u in the exponential changes the range of T_c only by a small amount. So effectively T_c lies in the range $0.2 \leq T_c \leq 0.4$. Putting back units, and defining the *boundary* temperature⁷ \mathcal{T} as $\mathcal{T} \equiv \frac{g'(u_h)}{4\pi\sqrt{h(u_h)}}$, our analyses reveal:

$$\frac{0.91}{L^2} \leq \mathcal{T}_c \leq \frac{1.06}{L^2} \quad (3.5)$$

which is the range of the melting temperatures in these class of theories for heavy quarkonium states. Since the temperatures at both ends do not differ very much, this tells us that the melting temperature is inversely related to the asymptotic AdS radius in large N thermal QCD.

4. Discussions and conclusions

In this work we attempted a small step towards the understanding of the melting temperatures of heavy quarkonium states in a class of large N thermal QCD that have well defined UV behaviors without Landau Poles and UV divergences of the Wilson loops. Our analyses reveal two interconnecting results: any large N gauge theory with N_f fundamental quarks ($N_f \ll N$) that is away from conformality with atmost one UV fixed point⁸ should always have a mass gap and, consequently at

⁷See sec. (3.1) of [2] for details.

⁸This also means a good UV behavior with degrees of freedom increasing monotonously with energy scales.

a certain temperature T_c , the heaviest quarkonium states in such a theory should show melting. Such a conclusion seems to be true for generic non-supersymmetric theories, and it would be interesting to extend this to the supersymmetric cases.

The melting temperatures⁹ that we get for various UV completions seem not to differ too much from each other. This may mean that there is some underlying universal behavior of heavy quarkonium states in large N theories with good UV behaviors at strong 'tHooft couplings. An interesting related study would be that of the behavior at weak 'tHooft couplings. There, there are also Hagedorn states that would come in because of the unsuppressed string modes. Details on this will be presented elsewhere.

Acknowledgements

It's our pleasure to thank Sumit Das, Juan Maldacena, Guy Moore and Mark VanRaamsdonk for helpful comments. This work was supported in part by the Natural Sciences and Engineering Research Council of Canada.

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⁹Note that their dependences on the 'tHooft coupling are consistent with what we would expect.

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